Flag-manifold sigma model: Phase structure, Anomaly, and Semiclassics (1)

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Refs: 1805.11423 with Tin Sulejmanpasic, 1812.02259 with Masaru Hongo, Tatsuhiro Misumi

Motivation: SU(N) anti-ferromagnetic spin chains

$SU(2)$: (1+1)d anti-ferromagnetic system has following features.

- Anti-ferro ($\uparrow\downarrow\uparrow\downarrow\cdots$) $\Rightarrow$ Spinon is "relativistic" $E = |p|$.
- For $|S| \to \infty$, local spin motion becomes classical. 2d $\mathbb{C}P^1$ ($= SU(2)/U(1)$) model with $\boldsymbol{\theta = (2S)\pi}$.
- Integer S: mass gap. Half-integer S: gapless. (Haldane '83)

What is an $SU(N)$ generalization?

Each spin is now in $SU(N)$ rep, e.g. the p-box symmetric rep.

Sigma model at $p \to \infty$: (Bykov '12, Lajkó, Wamer, Mila, Affleck '17)

$$\mathcal{L} = \sum_{\ell=1}^{N} \left[-\frac{1}{2g^2} \left| (\mathrm{d} + \mathrm{i}a_\ell)\boldsymbol{\phi}_\ell \right|^2 + \frac{\mathrm{i}\theta_\ell}{2\pi} \mathrm{d}a_\ell + \cdots \right], \tag{1}$$

where $U = [\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_N] \in SU(N)$, a_ℓ are $U(1)$ gauge fields, and

$$\theta_\ell = \frac{2\pi p}{N} \ell. \tag{2}$$

Symmetry

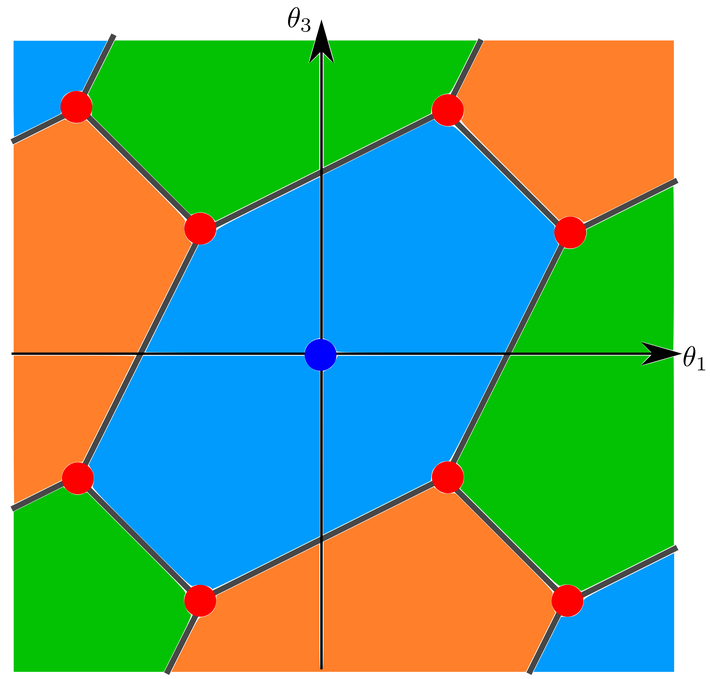
- $SU(N)/\mathbb{Z}_N$ spin rotation, $U \to V \cdot U$.
- Lattice translation $\Rightarrow$ $\mathbb{Z}_N$ permutation at $\theta_\ell = \frac{2\pi p}{N}\ell$,

$$\boldsymbol{\phi}_\ell \mapsto \mathrm{e}^{2\pi\mathrm{i}(\ell+1)/N} \boldsymbol{\phi}_{\ell+1}, \quad a_\ell \mapsto a_{\ell+1}. \tag{3}$$

- Time reversal T : $\boldsymbol{\phi}_\ell(x,t) \mapsto \overline{\boldsymbol{\phi}}_\ell(x,-t)$.
- C_k: $\boldsymbol{\phi}_i \mapsto (-1)^N \overline{\boldsymbol{\phi}}_{-i-k}$ and $a_\ell \mapsto -a_{-\ell-k}$ for certain θ_ℓ's.

Anomaly matching

Anomaly matching predicts a rich phase diagram ($N = 3$):



- Different colors: Different SPT states of $SU(3)/\mathbb{Z}_3$.
- Black solid lines: C-broken states.
- Red points: $SU(3)_1$ WZW CFT, or $\mathbb{Z}_3$-broken states.

As an example of anomaly, let us pick up a red point.

We introduce the $SU(3)/\mathbb{Z}_3$ background gauge field (A, B), where B is $\mathbb{Z}_3$ two-form gauge field.

$$(\mathbb{Z}_3)_{\text{perm.}} : Z[(A,B)] \to \exp\left(\mathrm{i}p \int B \right) Z[(A,B)]. \tag{4}$$

The gauged partition function acquires an anomalous phase, and nontrivial IR physics must be there for $p \notin 3\mathbb{Z}$.

Flag-manifold sigma model: Phase structure, Anomaly, and Semiclassics (2)

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Flavor-twisted S^1-compactification and anomaly

S^1-compactification of the spacetime gives an IR cutoff L^{-1}, which can stop the running of coupling.

Naive, thermal one, however, spoils all the interesting properties predicted by anomalies. No SSB at high temperatures.

Flavor-twisted S^1-compactification

We, instead, impose the twisted b.c.,

$$\boldsymbol{\phi}_\ell(x,\tau+L) = C\cdot\boldsymbol{\phi}_\ell(x,\tau), \tag{1}$$

with $C=\mathrm{diag}(1,\omega,\dots,\omega^{N-1})$, $\omega=\exp(2\pi\mathrm{i}/N)$.

Under this compactification, the mixed anomaly with $SU(3)/\mathbb{Z}_3$ persists at any sizes of L. (Tanizaki, Misumi, Sakai, '17)

This is because there is a $\mathbb{Z}_N$ symmetry acting on Polyakov loops under the twisted b.c.,

$$P_\ell = \exp\left(\mathrm{i}\int_0^L a_\ell\right) \to \omega P_\ell. \tag{2}$$

$\mathbb{Z}_N$ two-form gauge field B in 2d becomes the gauge field for this symmetry,

$$B = B^{(1)}\wedge\frac{\mathrm{d}\tau}{L}, \tag{3}$$

and thus the anomaly is still present in the compactified theory:

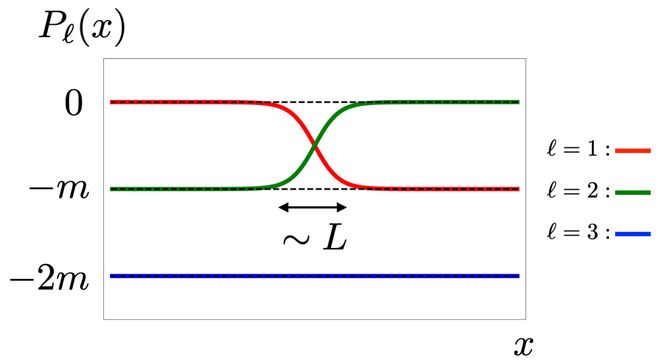
$$(\mathbb{Z}_N)_{\text{perm.}} : Z_{\text{twisted}}[B^{(1)}] \to \exp\left(\mathrm{i}p\int B^{(1)}\right) Z_{\text{twisted}}[B^{(1)}]. \tag{4}$$

Fractional instanton

BPS bound for the kinetic term (N=3):

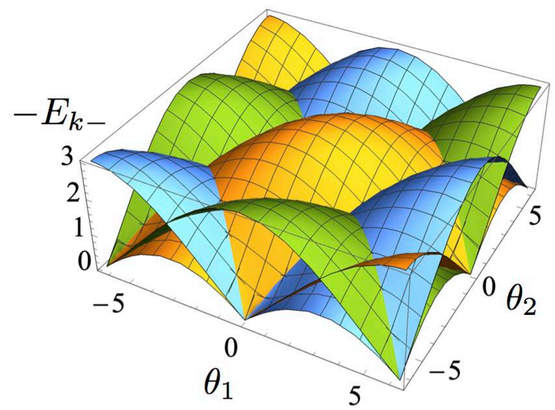
$$\sum_{\ell=1}^{3}\int|(\mathrm{d}+\mathrm{i}a_\ell)\boldsymbol{\phi}_\ell|^2 \geq \frac{2\pi}{2g^2}(|Q_1|+|Q_2|+|-(Q_1+Q_2)|). \tag{5}$$

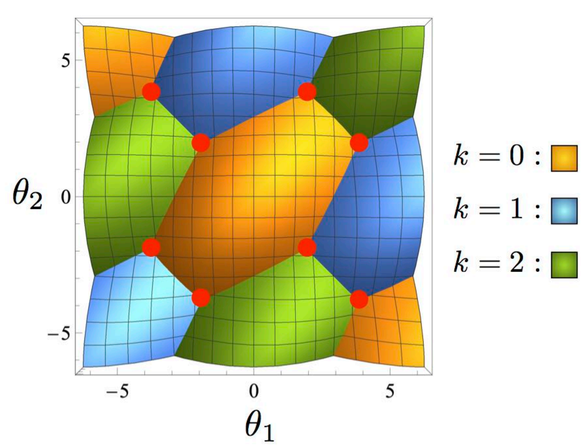
Fractional instanton ($Q_1=-Q_2=1/3$) for the twisted b.c.:



Semiclassical analysis with flavor-twisting

Dilute-gas approximation of fractional instantons:





Remarkable similarities between this figure and the conjectured phase diagram for 2d theory.